3.7 Complex Zeros ; Fundamental Theorem of Algebra

Conjugate Pairs Theorem:

Let $f(x)$ be a polynomial whose coefficients are real #'s. If $r = a+bi$ is a zero of f, then the complex conjugate $\bar{r} = a-bi$ is also a zero of f.

- A polynomial of odd degree w/real coefficients has at least one real zero.

Ex. 1

A polynomial f of degree 5 whose coefficients are real numbers has the zeros $1, 5i,$ and $1+5i$. Find the remaining two zeros.

- Since complex zeros appear as a conjugate pair, it follows that $\boxed{-5i \ \& \ 1-5i}$ are the remaining two zeros (conjugates of $5i$ & $1+5i$)

Ex. 2

Find a polynomial f of degree 4 whose coefficients are real #'s and that has the zeros of $1, 1, -4+i$

$$(x-1)^2[x-(-4+i)][x-(-4-i)] = \boxed{(x-1)^2(x^2+8x+17)}$$

Factoring complex conjugates in $a+bi$ form $\rightarrow \boxed{x^2 + -2ax + (a^2+b^2)} \rightarrow a=-4, b=1$

$\boxed{\text{p.237 \#19}}$ Deg = 5 ; zeros = 2, -i, 1+i

* $x^2 - 2ax + (a^2 + b^2)$ $\longrightarrow$ $\boxed{(x-2)(x^2+1)(x^2-2x+2)}$

if in a+bi form

Ex. 2 (continued)

b.) graph to verify your result

- At most, $n - 1 = 4 - 1 = \underline{3}$ turning points
- For large $|x|$, the graph will behave like $y = x^4$
- A repeated real zero at $x = 1$, so the graph will touch at $x = 1$.
- only x-intercept is at $x = 1$

(graph on your calcs to see the above scenarios)

Ex. 3

Find the complex zeros of $f(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$

1. Deg = 4, so f will have 4 complex zeros

$\hookrightarrow$ reminder: every real zero can be written as a complex zero (in that case, the $b = 0$, in $a + bi$ form)

2. graph on calc + see one of the zeros is $x = -2$, so $x + 2$ is a factor of f.

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & 25 & 45 & -18 \\ & 0 & -6 & 2 & -54 & 18 \\ \hline & 3 & -1 & 27 & -9 & 0 \end{array}$$

$(x+2)(3x^3 - x^2 + 27x - 9)$

- $\frac{1}{3}$ is another zero when looking at the graph + using 2nd trace $\rightarrow$ zero

$\left(x - \frac{1}{3}\right) = (3x - 1)$

$$\begin{array}{r|rrrr} 1/3 & 3 & -1 & 27 & -9 \\ & 0 & 1 & 0 & 9 \\ \hline & 3 & 0 & 27 & 0 \end{array}$$

$(x+2)(3x-1)(3x^2+27)$

$3(x+2)(3x-1)(x^2+9)$

$\boxed{\text{zeros} \rightarrow x = -2, 1/3, \pm 3i}$

p. 237 # 27

$h(x) = x^4 - 9x^3 + 21x^2 + 21x - 130$; zero → $3 - 2i$

$a = 3, b = -2$

one of the Factors must be:

$x^2 - 6x + 13$ [using $x^2 - 2ax + (a^2 + b^2)$]

$x^2 - 3x - 10$ → another factor

$$
\begin{array}{r}
x^2 - 3x - 10 \\
x^2 - 6x + 13 \overline{\big) x^4 - 9x^3 + 21x^2 + 21x - 130} \\
-\ \underline{x^4 - 6x^3 + 13x^2} \\
-3x^3 + 8x^2 + 21x \\
-\ \underline{-3x^3 + 18x^2 - 39x} \\
-10x^2 + 60x - 130 \\
(-)\ \underline{-10x^2 + 60x - 130} \\
0
\end{array}
$$

Factored Form $(x^2 - 6x + 13)(x^2 - 3x - 10)$

$(x^2 - 6x + 13)(x - 5)(x + 2)$

zeros → $x = 3 \pm 2i$ $\quad x = 5 \quad x = -2$

remaining zeros